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Stochastic Modeling of Particle Motion along a Sliding Conveyor

Kevin Cronin, Muammer Catak, Sean Dempsey, and Remi Ollagnier
Dept. of Process and Chemical Engineering, University College Cork, Ireland

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The sliding conveyor consists of a plane surface, known as the track, along which particles are induced to move by vibrating the bed sinusoidal with respect to time. The forces on the particle include gravity, bed reaction force and friction. Because friction coefficients are inherently variable, particle motion along the bed is erratic and unpredictable. A deterministic model of particle motion (where friction is considered to be known and invariant) is selected and its output validated by experiment. Two probabilistic solution techniques are developed and applied to the deterministic model, in order to account for the randomness that is present. The two methods consider particle displacement to be represented by discrete time and continuous time random processes, respectively, and permits analytical solutions for mean and variance in displacement versus time to be found. These are compared with experimental measurements of particle motion. Ultimately this analysis can be employed to calculate residence-time distributions for such items of process equipment. © 2009 American Institute of Chemical Engineers *AIChE J*, 56: 114–124, 2010
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Introduction

The vibrating conveyor consists of a plane surface, known as the track along which particles are induced to move by vibrating the track sinusoidal with respect to time. The amplitude and frequency of the oscillation together with the direction of the oscillation (inclined at some angle to the normal) are the design parameters of the conveyor. Such conveyors can be employed as stand alone items of equipment to achieve material transport, or alternatively, they can be integrated into process equipment and used to obtain a certain residence time in continuous processing. Two modes of operation are slide or throw: in the former, the conveyed material always remains in contact with the track while for the latter; contact is lost during the flight phase of the motion. Slide operation is preferred where the conveyed product is delicate and would be damaged by impact or where noise (and dust generation for the conveying of bulk powder) must be avoided. This mode also covers the application where the conveyor acts as a feeder. A simple view of the sliding mode of operation is that the conveyed material is carried forward by the frictional effect between itself and the track of the conveyor during the forward stroke, and depends on inertia to be left in the forward position as the track returns for the next stroke.[1] This article restricts itself to the analysis of the sliding mode of operation. In the industrial situation either an amorphous bed of granular material or a large number of individual discrete objects can be conveyed. For this work, the focus is on the motion of a single object on the conveyor or (which is the same thing) multiobject conveying where there is no interaction between the objects. Finally it is assumed that the conveyor track is horizontal and not operating at an incline.

Over the years a variety of approaches have been presented to model the motion of the conveyed particle for the general vibrating conveyor and more particularly for the sliding mode of operation.[2,3,4,5,6] In all cases the motion of the particle or material on the conveyor is predicted to be

Correspondence concerning this article should be addressed to M. Catak at muammer.catak@gmail.com

deterministic where for given settings of adjustable conveyor parameters (principally vibration amplitude and frequency), a given motion of the particle (usually quantified by average conveying velocity) results. However, measurements of the behavior of particles on sliding conveyors reveal that the motion can be quite erratic both for a given particle and between different particles. The dynamics of the conveyor track itself is entirely deterministic and is defined by the settings of the amplitude, frequency and direction of oscillation. The particle, resting on the track, is subject to two additional forces; gravity and friction. The latter is defined by the static and kinetic coefficients of friction between the conveyed particle and the track. As the only input variable to particle motion that can meaningfully exhibit randomness or unpredictability is friction, so any variability in particle motion can be considered to arise from variability in friction between the particle and track.

Friction coefficients are inherently unpredictable. Actual coefficients for a given situation will depend on the exact nature of the contacting surfaces; a variation of 25 to 100% from stated values could be expected in an actual application depending on prevailing conditions of cleanliness, surface finish, pressure, lubrication and relative velocity. Even with delicate measurements, the coefficients are rarely known to as greater precision than $\pm$ 10%.[7] To represent the random dispersion in friction, the coefficient of friction between a particle and a surface can be treated as a random function of position. Previously, in analyzing the avalanche problem for the friction driven motion of bodies on an inclined slope, Lima et al.[8] assumed that friction was due to the existence of random contact points between the surfaces. Therefore, the friction coefficient was modeled as a rapidly varying function of particle position on the plane. The authors made the further assumption that the distribution of contact points was uncorrelated along the length scale of interest, in effect treating the friction as a continuous white noise process to enable an analytical solution to be obtained for the distribution in the stopping distance. The positional dependence of friction has been used elsewhere in the literature,[9,10] to explain the dynamics of moving bodies where friction has an influence.

An alternative approach to using random process theory in the analysis of such a system is to consider the exhibited behavior as chaotic that is, the seemingly unpredictable nature of the output is in fact deterministic and stems from the nonlinear nature of the governing equations of motion (more particularly an extreme sensitivity to the initial conditions). Tufillaro[11,12,13] has examined the inelastic bouncing of a ball on a vertically vibrated bed, and reported on the chaotic behavior in the motion of the ball that was observed for certain settings of the system. Such a model is in principle not far removed from the throw mode of motion on a conveyor where the vertical component of conveyor bed acceleration is enough so the particles lose contact with the bed and forward motion is primarily by bouncing. According to this approach the seeming randomness in the displacement vs. time histories of the particles may not necessarily arise from noise in particle/bed friction, but rather is inherent in the dynamics of the process. In a related area, Behringer[14,15] and others have studied the motion of dense granular materials subject to vibration and other modes of excitation, and where surface friction had an important role. The intention was to obtain a general statistical description of the macroscopic physical properties and ordering of such systems. They found that by analyzing contact-force distributions, it was possible to predict equilibrium structures for the material. However, unlike the work of our article, the focus was on multiparticle systems and generally examining the physics of a situation where there is an asymptotic equilibrium.

The objective of this work is to investigate the nature of the randomness in the motion of particles on a sliding conveyor and to develop probabilistic models of the phenomenon. Output from models will be compared against experimental data to determine the validity of each method. Ultimately such work can provide a phenomenological basis for understanding and predicting residence time distribution for objects in such units.

Deterministic Model of Particle Displacement

The model and terminology of Nedderman and Harding, (which was based on the earlier approach of Booth and McCallion), will be taken to define deterministic particle motion as it was developed specifically for the analysis of an individual particle on a sliding conveyor. The authors applied Newton's second law to the particle to enable its displacement per period of oscillation, w as a function of the input parameters of amplitude of excitation, a, period of excitation T, angle of excitation γ, and coefficient of friction f to be calculated. Thus, w is the net amount that the particle slides forward for every cycle of oscillation of the track; it will be termed the unit displacement in this article. The problematic issue is that particle unit displacement is a discontinuous function of the operational parameters depending on their relative magnitudes. Nedderman and Harding demonstrated that during each cycle of oscillation, the particle is sequentially slipping forward with respect to the track, slipping backward or remaining stationary on the track for part of the cycle time. They identified four possible regimes of operation depending on the setting of system parameters; each regime has its own sequence of phases of particle motion. For the input conditions used in this work the most common regime (although not the only one) was the stationary, then slipping forward (positive), stationary again and then slipping backward (negative) regime. This is termed the SPSN mode. The distance of forward slip is termed w_P and backward slip, w_N and the net forward movement per vibration period is the difference between the two. For a given setting of amplitude, period, and angle of excitation (conveyor operational parameters) and a known coefficient of friction between the particle and track, particle unit displacement per vibration time period, w is analytically expressed as

$$w = w_P - w_N = a\lfloor(\sin\gamma + f\cos\gamma)F_1 + (\sin\gamma - f\cos\gamma)F_2\rfloor \tag{1}$$

where the dimensionless factors F_1 and F_2 are themselves implicit functions of conveyor parameters (vibration amplitude, period and angle), defined and explained in the original Nedderman and Harding article. In that article, particle motion is quantified by an (average) conveying velocity, v which is defined as

$$v = \frac{w}{T} \tag{2}$$

This velocity is not the actual or instantaneous speed of the particle; rather an overall measure of its progress that can be used to calculate throughputs and capacity of such items of equipment. The following assumptions were used or implied in the analysis: all particles are identical with respect to size and shape; there is no interparticle interaction or contact between particle and track side wall; conveyor track motion is perfectly sinusoidal; particles are treated as a point mass moving in a straight line parallel to conveyor track and with no rotational motion; air drag is negligible, and, hence, friction and gravity are the only active forces. Furthermore, Nedderman and Harding demonstrated that while the motion of the particle depends on both the static and kinetic coefficients of friction, the resultant displacement per period, w is primarily determined by the kinetic coefficient.

Stochastic Modeling Approaches

According to the Nedderman and Harding approach, the friction coefficient is treated as a simple scalar quantity quantified by a single value (ignoring the distinction between its static and kinetic magnitudes) for a given particle and conveyor track surface. As it is constant, then unit displacement per time step, w should be the same for every successive vibration period (assuming conveyor settings are constant) and the same for repeated experiments. However, careful measurements demonstrate that unit displacement, w is not constant but can vary unpredictably with time. Figure 1a illustrates two typical realizations of successive unit displacements vs. time; the corresponding constant level of w that the model expects is also shown (as the horizontal line). Each realization corresponds to a single hypothetical trial or experiment. Figure 1b gives the resulting cumulative particle displacement vs. time for each case. The figures indicate that w is not an invariant constant but rather exhibits dispersion. Furthermore, unit displacement, w cannot be modeled as a random variable (taking a different value from trial to trial but constant within a trial) as it changes within any given trial, as well as being different for separate trials. Hence, it must be considered as a random process and an appropriate model found for it; an analysis based on an autoregressive or colored noise process (which is more physically realistic than considering it as white noise) would seem appropriate.[16]

The underlying cause of the fluctuation in w must be that the friction coefficient varies randomly in space that is, along the length of the conveyor track in this case. From a physical perspective, the friction coefficient would be expected to vary continuously with position along the conveyor bed and be classified as a one-dimensional (1-D) random field.[17] However, a continuously varying friction coefficient would not be compatible with the Nedderman and Harding model of particle motion where friction must at least be constant over a length corresponding to a single period of vibration so a value for unit displacement w can exist and be calculated. Hence, the friction coefficient can be treated as remaining constant over the distance w, and then being randomly different for successive values of unit displacement. Two approaches based on the measured statistics of particle unit displacement will be utilized to examine the variability in particle motion. In the first method, particle unit displacement is treated as a discrete random sequence with particle displacement (i.e., cumulative displacement) the sum of the individual incremental displacements that have occurred. In the second approach, average particle velocity will be analyzed as a continuous time, first-order, autoregressive process as opposed to an invariant scalar quantity. Particle displacement is the integral of this with respect to time, and its statistics can be found by applying the theory of continuous random processes. Each method requires certain assumptions to be made about the nature of the variability in the process, and the validity of these can be tested by comparison with experimental output and the predictions of the other approach.

Particle unit displacement as a discrete time autoregressive process

For this approach, the change in particle (cumulative) displacement with time can be viewed as a discrete time random process where the time step is the period of vibration T. Particle displacement can be examined at any time that is an integer multiple of the time step and will be sum of the individual displacements per time step that have taken place up to that point

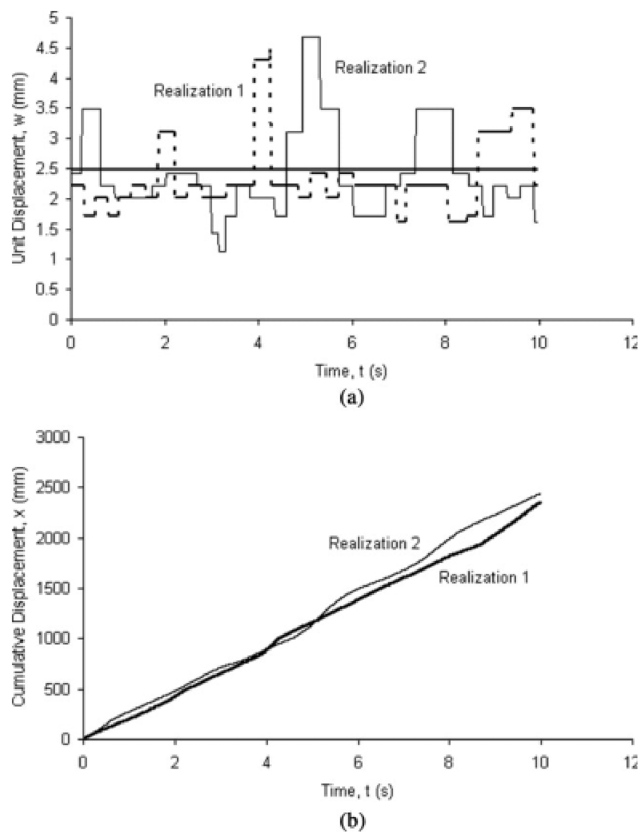

Figure 1. a. Realization of unit displacement vs. time; b. realization of cumulative displacement vs. time.

$$x_{n+1} = x_n + w_{n+1} = \sum_{i=0}^{n+1} w_i \quad (3)$$

where x_n is the position of the particle after n sequential time steps of duration T. For this analysis, unit displacement w is taken to be described as a sequence of correlated and identically distributed random variables (i.e., random values drawn described by the same probability distribution with presumed known statistics). It will be defined by a mean level, a variance and an autocorrelation parameter. Values of it can be generated using the following scheme

$$w_{n+1} = \mu_w + \rho_W(w_n - \mu_w) + z_{n+1} \quad (4)$$

where μ_w is the mean value of unit displacement (which can be predicted using equation 1 from the deterministic model), and ρ_w is the lag 1 autocorrelation coefficient for this unit displacement. The term z is the random component of w taken to be a normally distributed, iid process with zero mean and variance σ_z^2 given by

$$\sigma_z^2 = \left(1 - \rho_w^2\right)\sigma_w^2 \quad (5)$$

The magnitudes of these statistics (σ_w and ρ_w) and the assumption of normality can be found or checked from experimental analysis. Combining Eqs. 3 and 4 allows the particle displacement after n time steps to be defined as

$$x_n = \sum_{i=0}^{n}\left[\left(1 - \rho_w^i\right)\mu_w + \rho_w^i w_0 + \sum_{j=1}^{i} \rho_w^{i-j} z_j\right] \quad (6)$$

Particle unit displacement, w is a first-order autoregressive AR(1) (or Gauss-Markov) process and is asymptotically stationary. An AR(1) process describes the case where the current magnitude of the variable of interest depends only on its value at the immediately previous time step plus some random component. Cumulative displacement, x is a random increment process and is nonstationary; alternatively it can be considered as an integrated autoregressive AR(1,1) process whose first difference is an AR(1) process.[18] Mean particle displacement after n time steps, μ_X will be given as

$$\mu_X = n\mu_w = \frac{\mu_w}{T} t \quad (7)$$

showing mean particle cumulative displacement is the sum of the mean value of the unit displacements that have taken place. Mean displacement increases linearly with time (or number of time steps). Variance in particle displacement will be

$$\sigma_x^2[n] = \sigma_w^2\left(\frac{1-\rho_w^n}{1-\rho_w}\right)^2 + \left(1-\rho_w^2\right)\sigma_w^2\left(\frac{\rho_w^{2n+1} - \rho_w^{2n} - 2\rho_w^{n+2} + 2\rho_w^n + n\rho_w^3 - n\rho_w^2 + 2\rho_w^2 - n\rho_w - \rho_w + n - 1}{\left(\rho_w^3 - \rho_w^2 - \rho_w + 1\right)\left(\rho_w - 1\right)^2}\right) \quad (8)$$

As the number of elapsed time steps, n increases so does variance in displacement indicating that it is unbounded. Variance and its evolution with time is seen to be very sensitive to the magnitude of the autocorrelation coefficient for unit displacement, ρ_w. This can be expected to lie between 0 and 1 and it is instructive to examine the behavior of the system at these two extreme or limit values. As ρ_w goes to zero, then from Eq. 4 and 5, w becomes uncorrelated or discrete time white noise. Particle displacement x is then an independent increment process and its variance should rise linearly with time. This can be verified by noting

$$Limit\ \sigma_X^2[n] as \quad \rho_w \to 0 = n\sigma_w^2 \quad (9)$$

As ρ_w approaches unity, then again from Eqs. 4 and 5, w becomes a simple random variable invariant from time step to time step within an individual trial, but different from trial to trial. Then by Eq. 8, the variance in displacement will rise quadratically with time or number of time steps. Again this is borne out by the expression for particle variance by noting

$$Limit\ \sigma_X^2[n] as \quad \rho_w \to 1 = n^2\sigma_w^2 \quad (10)$$

In reality, the autocorrelation coefficient, ρ_w must lie at some intermediate position between these two limits, and so variance will rise at a rate intermediate between a linear and quadratic fashion although the long-term asymptotic behavior will be linear.

Particle velocity as a continuous time autoregressive process

The actual motion of the particles is inherently discretized with a natural time step equal to the period of vibration T. Nonetheless viewed at time scales much longer than T, the motion could be considered to be continuous. This approach follows from the Nedderman and Harding deterministic model where despite the actual disjointed (stop/start) nature of the real motion, an average conveying velocity is associated with each combination of conveyor vibration settings; this velocity is the average slope of the curves shown in Figure 1b. In this section, particle velocity will now be taken to vary stochastically with time rather than being constant. From the definition of velocity of Eq. 2, the mean and variance in velocity can be related to the statistics of unit displacement

$$\mu_V = \frac{\mu_W}{T} \qquad \sigma_V^2 = \frac{\sigma_W^2}{T^2} \quad (11)$$

As velocity is being treated as a continuous first-order autoregressive process or colored noise (analogous to w in the discrete domain), its autocorrelation function will have a decaying exponential form

$$\rho_V(\tau) = e^{-\phi\tau} \quad (12)$$

where τ is the separation or lag time, and φ is the autoregressive factor whose inverse is the characteristic correlation time τ_C. The magnitude of φ can be estimated from knowledge of the lag-1 autocorrelation coefficient, ρ_w of measured successive values of w by equating the lag time, τ to the period T, to give

$$e^{-\phi T} = \rho_w \quad \Rightarrow \quad \phi = -\frac{\ln \rho_w}{T} \tag{13}$$

Particle cumulative displacement will be the integral of velocity with respect to time

$$x = \int_0^t v(t)dt \tag{14}$$

The solution of this stochastic differential equation enables the mean and variance in particle displacement to be obtained.[19] Mean particle displacement as a function of time will be (assuming initial mean displacement is zero)

$$\mu_X(t) = \mu_V t \tag{15}$$

which is the same result as Eq. 7. The variance in particle displacement as a function of time will be

$$\sigma_X^2(t) = \frac{2\sigma_V^2}{\phi^2}\left(\phi t - 1 + e^{-\phi t}\right) \tag{16}$$

The evolution of variance with time is again sensitive to the magnitude of the autoregressive factor φ (or its inverse τ_C). As successive values of velocity become less correlated ($\varphi \rightarrow \infty$ and $\tau_C \rightarrow 0$), velocity approaches a white noise process and variance in displacement will increases linearly with time. As successive values of velocity become more correlated ($\varphi \rightarrow 0$ and $\tau_C \rightarrow \infty$), variance in displacement will increases quadratically with time. Furthermore, examining the expression for variance in particle displacement for times much shorter than the correlation time τ_C, variance rises quadratically with time as can be seen from a power series expansion of the exponential term. For times much greater than the correlation time, variance will increase linearly with time as the exponential term decays to zero. Thus, overall, variance will increase at an intermediate rate between linear and quadratic with respect to time as found with the discrete case.

Materials and Methods

Figure 2 shows the experimental apparatus. The bed was made from 2 mm thick stainless steel plate with an overall length of 510 mm, and width of 120 mm. The active length of the conveyor was 400 mm. Four spring steel blades were placed at each corner to act as the guide springs. The blades were connected to the bed at their top and to the equipment base at their ends via aluminum blocks. These blocks had an inclined face matching the angle of excitation. To excite the bed, an electromagnetic drive using pulsed single-phase AC supply was used. The solenoid was bolted to a holding plate that was itself adjustable to ensure the solenoid was always perpendicular to the angle of oscillation. The solenoid was wired to a power source and signal generator via an amplifier. This permits the frequency of oscillation to be varied between 5 Hz and 50 Hz, and the amplitude to be adjusted within 1 mm and 10 mm. The particles employed in the experimental work were M4 stainless steel nuts, placed on their flats. Their selection was primarily motivated by the desire for experimental ease and the requirement for uniformity of particles.

A series of experiments were carried out to

- check the uniformity of the surface roughness of the bed and to check the homogeneity of the particles used in the test.
- measure the static coefficient of friction at various points along the track.
- quantify particle unit displacement to determine the statistics of the input variables to the modeling approaches.
- obtain repeated cumulative particle displacement versus time histories and compare the measured variability to that predicted by the different stochastic approaches.

Surface roughness measurements were taken at a number of positions (over 80) along the conveyor bed using a Mitutoyo Surftest stylus recorder. The friction coefficient can be expected to have some dependency on surface roughness, and the intention of this work was to eliminate any systematic effect that might be present and distort the analysis. A large sample of M4 stainless steel nuts was obtained and their mass and geometry (maximum distance across flats) measured with a mass balance and digital vernier, respectively. These data was later used to check that the kinematic behavior of individual nuts was not systematically related to their mass or width. Due to experimental limitations, only the static coefficient of friction between the nuts and conveyor track was measured. This was achieved by measuring the angle of static friction (required slope to initiate sliding of the nut along the conveyor). These measurements were carried out at a large number of locations along the centerline of the conveyor track.

To quantify the motion of the M4 nuts, a high-speed camera (AOS Technologies) and image analysis software (Midas) were used to accurately record displacement vs. time histories. Different recording intervals were employed depending on the particular test but the most common measurement interval was every 10 ms (0.01 s). Initially a number of different settings of the conveyor vibration parameters were employed to check the predictions of the deterministic

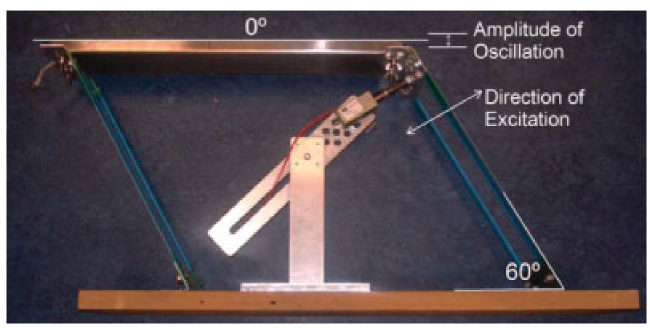

Figure 2. Experimental sliding conveyor.

[Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

model. Amplitude and frequency of vibration were varied although vibration angle was maintained at 60°. These parameters were adjusted within a relatively narrow range to ensure the results were comparable to the regimes identified by Nedderman and Harding, and to obtain smooth motion of the particles (no stalling) and continuous contact between particle and track. When determining the statistics of unit displacement and investigating the variability in particle motion, a single setting of the vibration parameters was used. This default setting was a vibration amplitude of 3.4 mm, frequency of vibration 6.25 Hz (Period of 0.16 s), and angle of vibration 60°. To quantify the variance in particle displacement, a particular nut was selected and its position noted every 0.16 s as it moved down the conveyor track. This was repeated 100 times (with the same nut), and mean and variance in position calculated. Note before each experiment, the track was wiped down with alcohol which was allowed to dry off. Each time the nut was directed down the centerline of the track. The sides of the track were raised to prevent the nuts falling off and any runs where the nuts impacted off the conveyor side walls were discounted.

Results

Particle and track characterization

The average surface roughness of the conveyor track was 0.17 microns, with a standard deviation of 0.047 microns. Most of the surface has a roughness value lying between 0.09 microns and 0.32 microns. There was no systematic pattern to the distribution in roughness. Thirty M4 nuts were used in the experiments as the conveyed particles. Mean particle mass was 0.86 g with a standard deviation of 0.015 g (a coefficient of variation of 1.7%). Mean width across flats was 6.86 mm with a standard deviation of 0.02 mm (a coefficient of variation of 0.3%). Both these sets of statistics with tightly controlled dispersion indicate that the particles can be considered to be drawn from a homogeneous population. Two scatter plots of the average conveying velocity of each nut vs. mass and dimension respectively revealed there was no dependency between them. Figure 3 displays the measured distribution in the static coefficient of friction between the particle and conveyor track in frequency histogram form. A chi-square test indicated that the distribution could be treated as Gaussian with a mean value of 0.294, and standard deviation of 0.051 (a coefficient of variation of 17%). This agrees with literature values for the static coefficient of friction between contacting stainless steel surfaces which can be expected to be between 0.31 and 0.41 depending on the degree of lubrication present.[20] Broadly speaking, the static coefficient of friction ranged from a minimum of 0.13 up to a maximum of about 0.48.

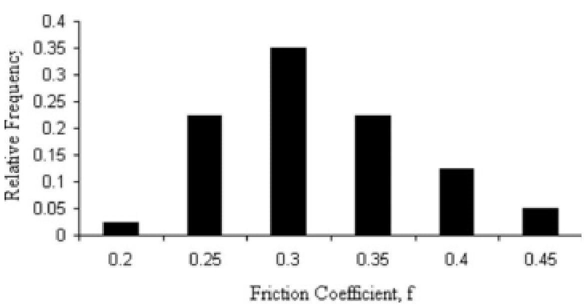

Figure 3. Frequency histogram of static coefficient of friction.

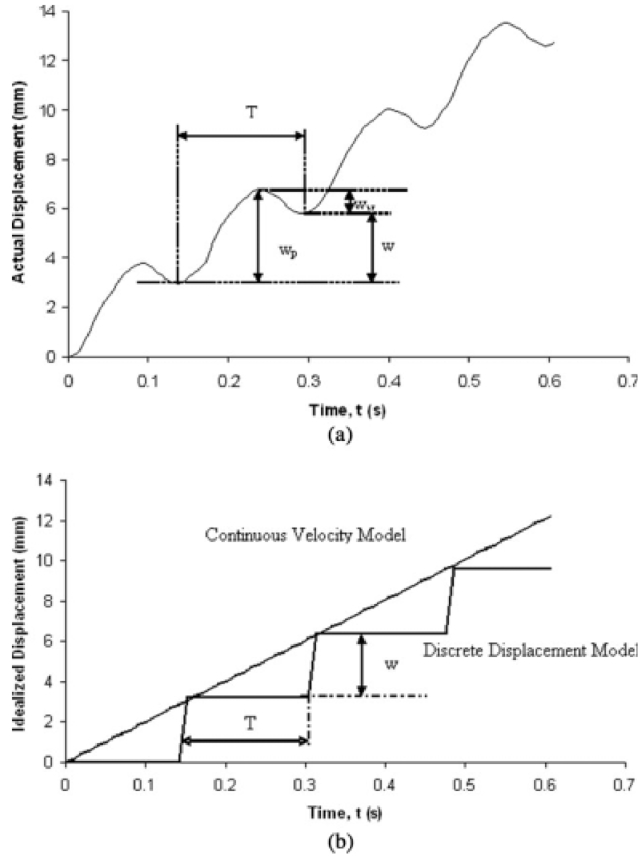

Figure 4. a. Actual particle displacement vs. time; b. idealized particle displacement vs. time.

Deterministic model validation

The output of the deterministic model was experimentally verified to ensure it captured the underlying physics accurately. Figure 4a shows actual particle displacement over four successive periods, T (each 0.154 s in duration) of excitation where the SPSN mode of motion prevailed. The forward, w_P and backward, w_N components of the motion are identified and clearly evident; their respective magnitudes are 4.1 mm and 0.85 mm, respectively giving a net displacement of 3.3 mm. The stationary phase is very short and cannot be distinguished. Over each single period, the particle is subject eight sequential accelerations (more precisely four accelerations and four decelerations), assuming the stationary-slipping forward-stationary-slipping backward (SPSN) mode of motion occurs. Unit displacement, *w* is the net forward advance of the particle per cycle, and not the actual displacement that has taken place. Figure 4b illustrates the idealized particle displacement vs. time history that the discrete and continuous modeling approaches use. It is clear from this that both stochastic methods of analysis of particle kinematics involve an idealized approach to the complex

Table 1. Validation of Deterministic Model

Setting	a mm	T s	w (experiment) mm	w (model) mm	Difference %
1	5.63	0.2	4.96	5.22	5.2
2	3.4	0.154	3.24	3.33	2.8
3	4.05	0.182	2.91	2.88	1.0
4	3.14	0.167	1.75	1.77	1.1

nature of the actual motion; however, as both methods have the objective of accurately predicting the statistics of the motion and not the motion itself, the approximation seems justifiable.

Table 1 gives the experimentally measured unit displacement and the predicted value from the model for four different settings of vibration parameters of the conveyor. Note for all the tests, the angle of excitation of the track, γ was 60°. In the calculation of w, the mean value of the coefficient of friction of 0.294 was used. The fractional difference between theory and experiment is between 1 and 5% demonstrating the applicability of the underlying deterministic approach (i.e., the Nedderman and Harding model) to motion of this type.

Statistical analysis of unit displacement

For the main vibration settings of amplitude 3.4 mm and frequency of 6.25 Hz (i.e., period of 0.16 s), the successive unit displacements, w were measured against time as the particles moved along the length of the conveyor and for a number of repeated trials. Figure 5 displays the distribution in the magnitude of w in frequency histogram form. From Figure 5, it is clear that there is significant dispersion in the magnitude of the unit displacement; values can range from as low as just above 1 mm up to values in excess of 5 mm. The distribution was also found to be Gaussian with a mean value of 3.2 mm, and standard deviation of 0.74 mm. Figure 6a illustrates the measured readings or sequence of successive unit displacements vs. time (over a time of 20 s). The first-order (lag 1) autocorrelation coefficient was found to be 0.92. The high value for the autocorrelation coefficient of 0.92 that underlies Figure 6a implies successive values of unit displacement are strongly correlated. This is more clearly seen in Figure 6b where unit displacement is plotted over a shorter time period and the time step or period, T of

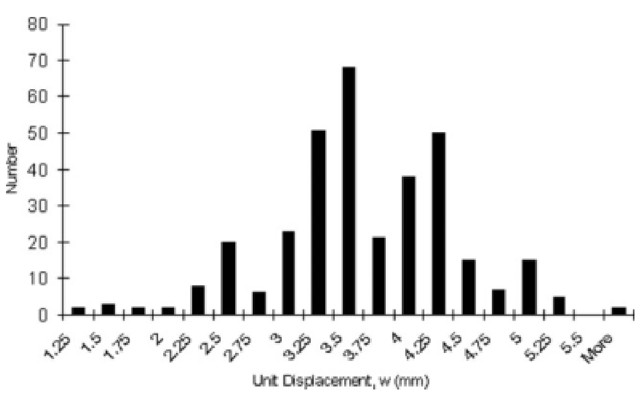

Figure 5. Frequency histogram of unit displacement.

0.16 s is indicated. Finally knowing the statistics of w, the corresponding statistics for particle velocity, v can be determined using Eqs. 11 and 13. Table 2 summarizes these statistics.

Analysis of statistics of particle displacement

Figure 7 illustrates six measured cumulative particle displacement vs. time histories showing that dispersion is significant. Each trace in the figure corresponds to the displacement history of a single M4 nut along the conveyor. The average slope in the displacement trace corresponds to the "conveying velocity" as defined in the deterministic model. It is clear that there is both a variation in the slope (velocity) between particles, and also over time a variation for the same particle. According to the deterministic model, all these displacement vs. time histories should exactly overlap and be identical. The presence of the high value for the autocorrelation coefficient for w is experimentally borne out as examining any displacement versus time history of an individual particle (in the figure) it can be seen that over a run, the average conveying velocity is relatively invariant with changes to the velocity being quite infrequent. The differences in conveying velocity between different runs or trials are far greater than those that occur within a specific run. Figure 8 shows simulated cumulative displacement vs. time histories for a number of runs as predicted by the discrete-time analytical model, the continuous-time analytical model and as experimentally measured. The general shape of the profiles from the analytical approaches is close to those

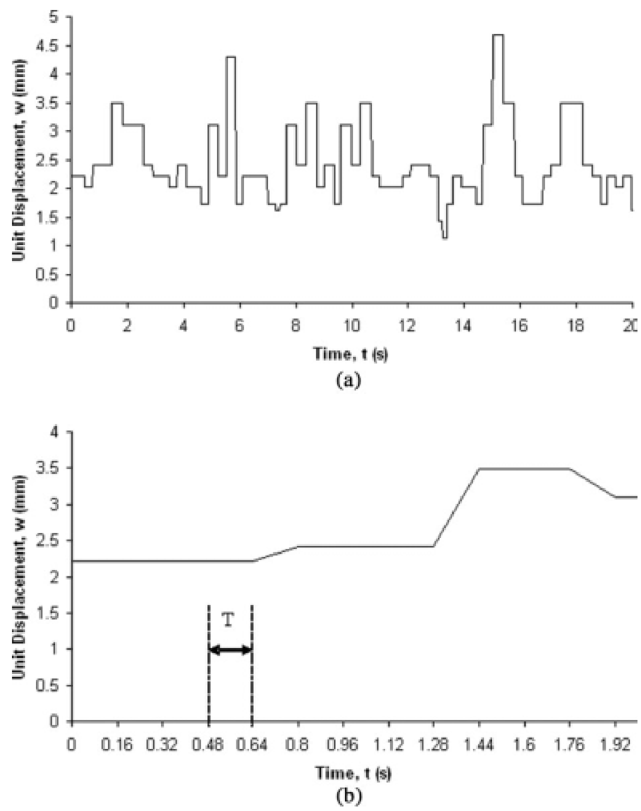

Figure 6. Measured profile of unit displacement vs. time.

Table 2. Statistical Parameters of the Input Variables

Variable		Mean (μ)	Standard Deviation (σ)	Auto-Correlation Parameter (P_w)	Auto-Correlation Parameter (φ)
Displacement	w	3.2 mm	0.74 mm	0.92	
Velocity	v	20.0 mm/s	4.63 mm/s		0.52 s^{-1}

experimentally found. Figure 9 displays simulated particle velocity vs. time profiles showing a slow variation with time consistent with the correlation time for the signal of just under 2 s. Average particle velocity is close to 20 mm/s although the model permits excursions down to 10 mm/s up to 30 mm/s.

Figure 10 compares the measured mean displacement vs. time and the predictions of both analytical approaches (Eqs. 7 and 15, respectively). Note both analytical approaches give identical predictions of mean displacement and their respective profiles are coincident. Mean particle displacement is predicted to rise linearly with time with a slope of 20 mm/s, which is equivalent to the displacement per time step of 3.2 mm. The experimental trace of mean displacement vs. time has a slight concave shape showing that (for these experiments) mean particle velocity increases marginally as the particles move along the track. Figure 11 compares the measured variance in particle displacement vs. time and the predictions of the analytical methods (Eqs. 8 and 16). Measured variance is significant; for instance after 10 s it is 630 mm^2 which corresponds to a standard deviation in displacement of 25 mm. With mean particle displacement at this time being 200 mm, the coefficient of variation in displacement is 0.125. Thus, in terms of reasonable minimum and maximum values, the slowest moving particle may have only travelled 150 mm in this time (mean minus two standard deviations), while the fastest particle may have achieved 250 mm. This is also borne out experimentally from the displacement histories of Figure 7. Alternatively, expressing variability in motion in terms of residence time (over a 200 mm distance), residence time could be expected to vary from 8 s to over 13 s. The agreement between both analytical approaches and measured variance is quite good and with both methods capturing its nonlinear rise with time. The discrete displacement model consistently underestimates the measured variance by a modest amount, and the continuous model gives a better prediction.

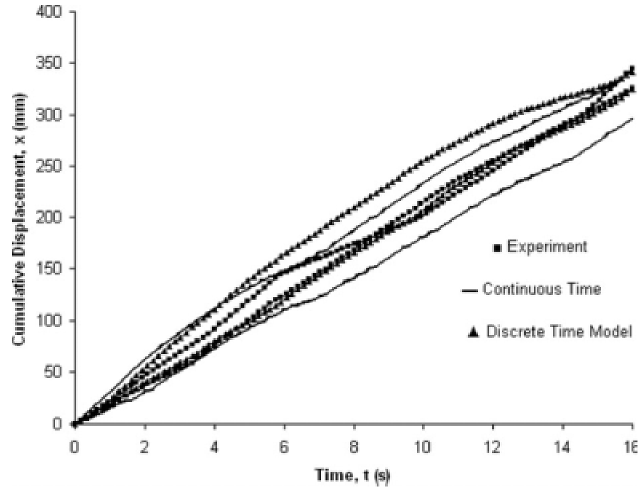

Figure 8. Particle cumulative displacement vs. time; experiment and models.

Discussion

The level of measured autocorrelation between successive values of unit displacement, ρ_w at 0.92 is close to unity (perfect correlation) and very high. If ρ_w had a value of 1, then w would be a random variable and variance in particle (cumulative) displacement would increase quadratically with time. At the other extreme, if the autocorrelation coefficient was 0, w would be equivalent to a discrete time white noise signal, and variance would increase linearly with time. Figure 12 illustrates the progression of variance in displacement over time from the discrete displacement approach with the autocorrelation coefficient of 0.92, and the limits when the autocorrelation coefficient is 0 and 1, respectively (all other input data being the same). Even though successive values of w are very strongly correlated (indeed almost perfectly correlated), the variance is considerably lower than that if w was treated as a random variable. Thus, to accurately predict the dispersion in particle motion, it is important to represent the nature of the variability in unit displacement correctly. Specifically at high levels of autocorrelation, predicted

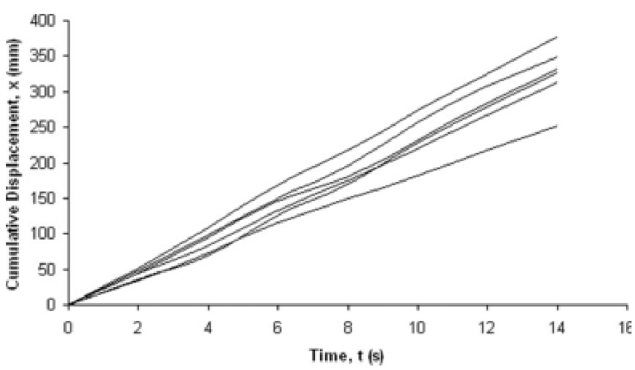

Figure 7. Experimental particle displacement histories.

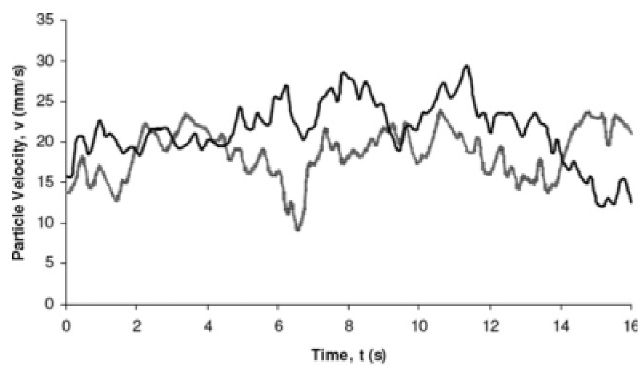

Figure 9. Predicted particle velocity vs. time (continuous model).

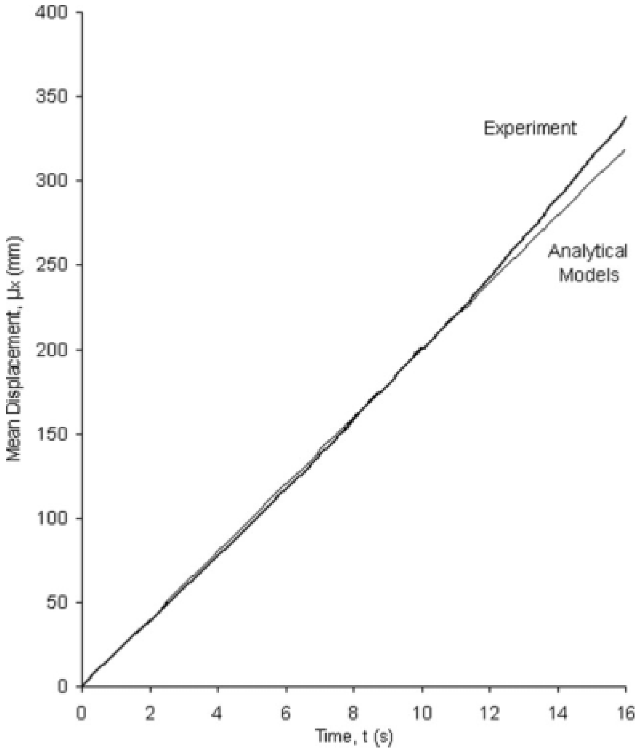

Figure 10. Experimental and predicted mean particle displacement vs. time.

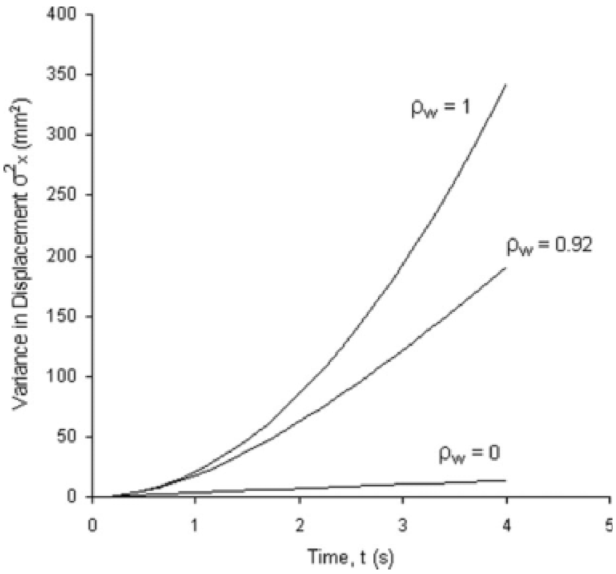

Figure 12. Evolution of variance in displacement versus time as affected by level of autocorrelation.

variance in displacement is very sensitive to the actual level of ρ_w; from the figure after 4 s of motion, the actual standard deviation in displacement is 13.8 mm ($\rho w = 0.92$), while it would be 18.5 mm (a third greater) if a ρ_w of unity was employed.

Another statistic of interest is the coefficient of variation of particle displacement and how it varies with time. The coefficient is the standard deviation in displacement divided by the mean value and provides a more useful estimate of the relative dispersion in particle position. Figure 13 plots this statistic vs. time from experimental data and from the continuous time model output. While the predictions are divergent at short times, at longer times they converge and both show that relative dispersion falls with longer conveying time. Examining the analytical expressions for variance in displacement (Eq. 16), and mean displacement (Eq. 15), it is clear that the coefficient of variation will asymptotically approach zero at very long times, although the rate of convergence is very slow. For industrial practice, this result suggests that longer conveyer lengths should be preferred where a tighter control over relative dispersion in particle displacement (and by extension residence time) is advantageous.

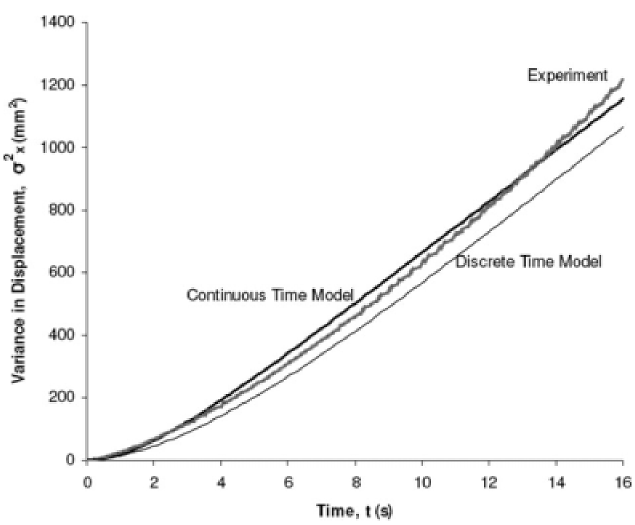

Figure 11. Experimental and predicted variance in particle displacement vs. time.

One drawback with the analysis that has been presented is that it models the measured variability in unit displacement, but does not provide an underlying explanation for the randomness in w. However, the assumption that friction between a particle and the track can be represented by a random field seems plausible, and lends itself as a basis for an explanation. Conceptually, dispersion in unit displacement can be related to the fluctuation in friction using the validated deterministic model. Using the Nedderman and Harding model, a relationship can be found between prevailing levels of friction and consequent unit displacement for any chosen conveyor settings. Hence, random variability in friction can be used to calculate the dispersion in unit displacement. A subsidiary Monte Carlo simulation study was carried out to verify the predictions of the random process analysis.[21] The measured dispersion in the coefficient of friction (shown in Figure 3) was sampled to obtain successive values of unit displacement from equation 1. Friction coefficient was considered as a random field, remaining constant over each calculated value of w, and then being randomly different for the next calculated value. Such an approach did not improve the prediction of variance in displacement (as the relationship between friction and displacement is both disjointed and extremely nonlinear and, furthermore, the static and kinetic values both have a role), but it does provide a

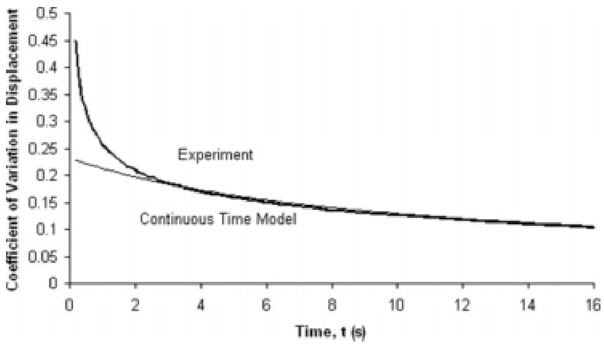

Figure 13. Coefficient of variation in particle displacement.

better rationale of the phenomenon. The alternative conjecture that the exhibited dispersion in particle displacement actually arises, not from noise in the friction between the particle and the bed, but from the inherent nonlinearity of the system cannot be discounted and warrants further study. This would involve an analysis of the equations of the deterministic model of particle motion to see if a chaotic response is possible for the same settings as used in this study.

Finally, it must be stated that this article analyses an idealized version of industrial conveying in that the motion of a single particle with regular geometry is studied. The real situation where a bed of amorphous granular material is conveyed is significantly more complex and requires a different approach. Questions such as the relationship between the friction coefficient and overall or mean conveying velocity are of interest to industrial users and have been partly addressed in the second article of Nedderman.[5] Real bulk granular materials have complex individual geometries with heterogeneous distributions in shape, size, asperity density etc. and characterization of their morphology is a prerequisite to understanding their kinetics.

Conclusions

The motion of particles on a sliding conveyor has been shown to be an inherently variable process. Two approaches to explain and predict the randomness in the motion of the particles have been developed and validated. Both these approaches adopt a simplified view of the actual motion of the particle in that it is accepted that the particle moves a certain distance, w in each time step. The actual displacement vs. time path *over a single vibration period* is not considered; using the simplified approach enables tractable random process theory tools to be applied to the problem. Experimentally measured successive unit displacements demonstrate that they are not invariant, but vary randomly and exhibit correlation. This fact forms the basis of the both analytical approaches (one discrete and one continuous), and results from them agree with measured statistics of particle motion. The analysis shows that dispersion in particle cumulative displacement is very sensitive to the level of autocorrelation of successive unit displacements. Future work will involve examining multiparticle systems and accounting for the effect of interparticle collision on the motion. There is also the interesting possibility of approaching and modeling the system from the entirely different perspective of chaos theory. Knowledge of the behavior of variance in particle displacement vs. time can be transformed into a residence-time distribution for the particles on the conveyor system.

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Notation

a = amplitude of vibration, m
f = coefficient of friction
t = time, s
T = period of vibration, s
v = particle velocity, m/s
w = net displacement per period of oscillation (unit displacement), m
w = forward displacement per period of oscillation (unit displacement), m
w = backward displacement per period of oscillation (unit displacement), m
x = particle (cumulative) displacement, m
γ = angle of excitation °
μ = mean
ρ = autocorrelation coefficient (lag 1)
σ = standard deviation
τ = separation or lag time, s
τ_C = correlation time, s
φ = autoregressive factor s^{-1}

Literature Cited

1. Woodcock CR, Mason JS. *Bulk Solids Handling*. Glasgow: Hill; 1987.
2. Sloot EM, Kruyt NP. Theoretical and experimental study of the transport of granular materials by inclined vibratory conveyors. *Powder Technol*. 1996;87:203–210.
3. Erdesz K, Szalay A. Experimental study on the vibrational transport of bulk solids. *Powder Technol*. 1988;55:87–96.
4. Nedderman RM, Harding GHL. The flight-free vibrating conveyor: Part 1: Basis theory and performance analysis. *Trans IChemE*. 1990 Part A;68:123–130.
5. Nedderman RM, Harding GHL. The flight-free vibrating conveyor: Part 2: Stability analysis and criteria for optimal designs. *Trans IChemE*. 1990 Part A;68:131–138.
6. Booth JH, McCallion H. On predicting the mean conveying velocity of a vibratory conveyor. *Proceedings IMechE*. 1963;Part 1:521–538.
7. Arnell RD, Davies PB, Halling J, Whomes TL. *Tribology: Principles and Design Applications*. London: Macmillan; 1991.
8. Lima AR, Moukarzel CF, Grosse I, Penna TJP. Sliding blocks with random friction and absorbing random walks. *Phys Rev*. 2000;61:2267–2271.
9. McMillan AJ. A non-linear friction model for self-excited vibrations. *J Sound Vibr*. 1997;205:323–335.
10. Popovic MR, Goldenberg AA. Modeling of friction using spectral analysis. *IEEE Trans Robot Autom*. 1998;14:114–122.
11. Tufillaro NB, Albano AM. Chaotic dynamics of a bouncing ball. *Am J Phys*. 1986;54:939–944.
12. Mello TM, Tufillaro NB. Strange attractors of a bouncing ball. *Am J Phys*. 1987;55:316–320.
13. Tufillaro NB. Braid analysis of a bouncing ball. *Phys Rev E*. 1994;50:4509–4522.

14. Behringer RP, Karen ED, Majmudar TS, Sperl M. Fluctuations, correlations and transitions in granular materials: statistical mechanics for a non-conventional system. *Phil Trans R Soc*. 2008; 366:493–504.
15. Dutt M, Behringer RP. Effects of surface friction on a two-dimensional granular system: Numerical model of a granular collider experiment. *Phys Rev E*. 2007;75:021305.
16. Stark H, Woods JW. *Probability, Random Processes and Estimation Theory for Engineers*. NJ: Prentice-Hall; 1994.
17. Vanmarcke E. *Random Fields - Analysis and Synthesis*. Cambridge: MA, MIT Press; 1983.
18. Cryer JD. *Time Series Analysis*. Boston: Duxbury Press; 1986.
19. Gardiner CW. *Handbook of Stochastic Methods*. 2nd ed. Berlin: Springer; 1997.
20. Ashby MF, Jones DRH. *Engineering Materials 1, An Introduction to Their Properties and Applications*. 3rd ed. Oxford UK: Elsevier; 2005.
21. Nicolai BM, Scheerlinck N, Hertog MLATM. *Probabilistic Modeling (Chapter 9)*. In: *Handbook of Food and Biprocess Modeling Techniques*. Florida: Taylor & Francis Group; 2007.

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